

# Intersecting D-branes and Lifshitz-like space-time

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## Abstract

In a previous paper [1] we have shown how Lifshitz-like space-times (space-times having Lifshitz scaling with hyperscaling violation) arise from 1/4 BPS, threshold F-D $p$  bound state solutions of type II string theories in the near horizon limit. In this paper we show that similar structures also arise from the near horizon limit of 1/4 BPS, threshold intersecting D-brane solutions of type II string theories. Some of these solutions are standard (D $p$ -D( $p+4$ ) for  $p = 0, 2$ ) and some are non-standard (D $p$ -D( $p+2$ ) for  $p = 1, 2, 3$ ) including D2-D2', D3-D3' and D4-D4' solutions. The dilatons of these solutions in general run (except in D2-D4 and D3-D3' cases) and produce RG flows. We discuss the phase structures of these solutions. D2-D4 and D3-D3' in the near horizon limit do not produce Lifshitz-like space-time, but give AdS<sub>3</sub> spaces.

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# 1 Introduction

Lifshitz scaling symmetry, a nonrelativistic symmetry, arises as a possible symmetry in some condensed matter systems at the quantum critical point [2, 3]. As the system at this point is strongly coupled, it can be studied holographically by using the general idea of AdS/CFT correspondence [4] if a gravity dual, which asymptotes to a space-time with a Lifshitz scaling symmetry, can be found for such systems. Indeed such metrics were found in [5] as solutions of pure gravity theory coupled to matter. Inclusion of dilaton enlarges the domain of such scaling symmetry of the metrics [6, 7, 8, 9, 10, 11, 12, 13]. The string embeddings of this class of metrics were obtained in [14].

A more general class of scaling metrics (i.e., metrics with Lifshitz-like scaling, namely, a Lifshitz scaling with a dynamical critical exponent  $z$  and a hyperscaling violation exponent  $\theta$ ) in the infrared have been found [15] by using a general scaling argument, the logarithmic violation of the entanglement entropy and the null energy condition. Holographically they represent compressible metallic states with hidden Fermi surface [16]. The whole class of such scaling metrics are obtained as solutions to pure gravity theories coupled to both the dilaton and an abelian gauge field [9]. Aspects of holography and some string theory embeddings of these class of metrics have been obtained in [15, 17, 18, 19, 20]. In a previous paper [1] we have shown how such metrics arise from the near horizon limit of some unusual 1/4 BPS, threshold F-D $p$  bound state solutions of type II string theories. In this paper, we show that similar structures also arise from the near horizon limit of intersecting D-brane solutions of type II string theories. Some of these solutions are the standard 1/4 BPS threshold intersecting D $p$ -D( $p+4$ ) (with  $p = 0, 2$ ) solutions of type IIA string theory and some are non-standard 1/4 BPS threshold intersecting D $p$ -D( $p+2$ ) (with  $p = 1, 2, 3$ ) including D2-D2', D3-D3' and D4-D4' solutions of type II string theories. The dilatons for all these solutions (except D2-D4 and D3-D3') are non-constant and therefore produce RG flows. We discuss the phase structures for these solutions and find that the metrics in other phases also have similar scaling structures. The near horizon metric of D2-D4 and D3-D3' solutions do not have Lifshitz-like scaling symmetry, but have the structures of AdS<sub>3</sub>-spaces in both phases.

This paper is organized as follows. In section 2, we discuss the standard intersecting D $p$ -D( $p+4$ ) (for  $p = 0, 2$ ) solutions of type IIA string theory their near horizon limits, scaling structures and the phase structures. In section 3, we discuss the same for the non-standard intersecting D $p$ -D( $p+2$ ) (for  $p = 1, 2, 3$ ) solutions along with D2-D2', D3-D3' and D4-D4' solutions of type II string theories. Then we conclude in section 4.

## 2 Dp-D( $p+4$ ) and Lifshitz-like metrics

In this section we will show that the standard 1/4 BPS threshold intersecting Dp-D( $p+4$ ) solutions of type II string theories in the near horizon limit yield Lifshitz-like metrics. For  $p=1$ , we know that D1-D5 solution of type IIB string theory does not give Lifshitz-like space-time but gives  $\text{AdS}_3 \times \text{S}^3 \times \text{E}^4$  in the near horizon limit. So, we will consider only  $p=0, 2$  in the following. The string metric and the other field configurations for Dp-D( $p+4$ ) intersecting solutions have the forms (see for example, [21]),

$$\begin{aligned} ds^2 &= H_1^{\frac{1}{2}} H_2^{\frac{1}{2}} \left[ H_1^{-1} H_2^{-1} \left( -dt^2 + \sum_{i=1}^p (dx^i)^2 \right) + H_2^{-1} \sum_{j=p+1}^{p+4} (dx^j)^2 + dr^2 + r^2 d\Omega_{4-p}^2 \right] \\ e^{2\phi} &= H_1^{\frac{3-p}{2}} H_2^{-\frac{p+1}{2}} \\ A_{[p+1]} &= (1 - H_1^{-1}) dt \wedge dx^1 \wedge \dots \wedge dx^p, \quad A_{[p+5]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge \dots \wedge dx^{p+4} \end{aligned} \quad (1)$$

where the two harmonic functions have the forms  $H_{1,2} = 1 + Q_{1,2}/r^{3-p}$ .  $Q_{1,2}$  are the charges associated with Dp and D( $p+4$ ) branes. The Dp branes are along  $x^1, \dots, x^p$  and D( $p+4$ ) branes are along  $x^1, \dots, x^{p+4}$ .  $r$  is the transverse radial coordinate given by  $r = \sqrt{(x^{p+5})^2 + \dots + (x^9)^2}$ . Note from (1) that for both  $p=0, 2$ , dilaton  $\phi$  are not constant and we have put the string coupling  $g_s = 1$ .  $A_{[p+1]}$  and  $A_{[p+5]}$  are the RR form fields which couple to Dp branes and D( $p+4$ ) branes respectively<sup>3</sup>.

In the near horizon limit we approximate  $H_{1,2} \approx Q_{1,2}/r^{3-p}$ . Substituting this in the metric in (1) and further making a coordinate transformation  $r \rightarrow 1/r$  we get,

$$ds^2 = \sqrt{Q_1 Q_2} r^{1-p} \left[ -\frac{dt^2}{Q_1 Q_2 r^{4-2p}} + \frac{\sum_{i=1}^p (dx^i)^2}{Q_1 Q_2 r^{4-2p}} + \frac{\sum_{j=p+1}^{p+4} (dx^j)^2}{Q_2 r^{1-p}} + \frac{dr^2}{r^2} + d\Omega_{4-p}^2 \right] \quad (2)$$

Now introducing a new coordinate by the relation  $u^2 = r^{1-p}$  we can rewrite the metric in (2) and the other field configurations in (1) in terms of  $u$  as,

$$\begin{aligned} ds^2 &= \sqrt{Q_1 Q_2} u^2 \left[ -\frac{dt^2}{Q_1 Q_2 u^{\frac{4(2-p)}{1-p}}} + \frac{\sum_{i=1}^p (dx^i)^2}{Q_1 Q_2 u^{\frac{4(2-p)}{1-p}}} + \frac{\sum_{j=p+1}^{p+4} (dx^j)^2}{Q_2 u^2} + \frac{4}{(1-p)^2} \frac{du^2}{u^2} \right. \\ &\quad \left. + d\Omega_{4-p}^2 \right] \\ e^{2\phi} &= \frac{Q_1^{\frac{3-p}{2}}}{Q_2^{\frac{p+1}{2}}} u^{2(3-p)} \\ A_{[p+1]} &= -\frac{1}{Q_1 u^{\frac{2(3-p)}{1-p}}} dt \wedge dx^1 \wedge \dots \wedge dx^p, \quad A_{[p+5]} = -\frac{1}{Q_2 u^{\frac{2(3-p)}{1-p}}} dt \wedge dx^1 \wedge \dots \wedge dx^{p+4} \end{aligned} \quad (3)$$

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<sup>3</sup>Here and below the constant terms in the form-fields are added such that the solution is asymptotically flat. However, in the near horizon limit when we deal with asymptotically non-flat solutions, we ignore the constant terms in the form fields.

It is clear from the metric in (3) that under the scaling  $u \rightarrow \lambda u$ , the coordinates  $x^1, \dots, p$  and  $x^{p+1}, \dots, p+4$  scale differently if the part of the metric in square bracket has to remain invariant. So, we will discuss  $p = 0$  and  $p = 2$  cases separately.

## 2.1 $p = 0$ or D0-D4 case

In this case we observe from (3) that under the scaling  $t \rightarrow \lambda^4 t \equiv \lambda^z t$ ,  $x^1, \dots, 4 \rightarrow \lambda x^1, \dots, 4$ ,  $u \rightarrow \lambda u$ , where  $z$  in  $t$  transformation is called the dynamical critical exponent, the metric in the square bracket is invariant. However, the full metric is not invariant as there is a hyperscaling violation [9, 17, 16]. To find the hyperscaling violation exponent we need to perform a dimensional reduction of the theory on  $S^4$  and express the resulting metric in Einstein frame. The reduced metric in this case can be seen to transform as  $ds_6 \rightarrow \lambda^{1/2} ds_6 \equiv \lambda^{\theta/d} ds_6 \equiv \lambda^{\theta/4} ds_6$ , where  $\theta$  is the hyperscaling violation exponent and  $d$  is the spatial dimension of the boundary theory. We thus find that the near horizon limit of intersecting D0-D4 solution has a Lifshitz-like metric with  $z = 4$  and  $\theta = 2$ . The dilaton and the form fields also transform (see (3)) under the above scaling as,  $\phi \rightarrow \phi + 3 \log \lambda$ ,  $A_{[1]} \rightarrow \lambda^{-2} A_{[1]}$  and  $A_{[5]} \rightarrow \lambda^2 A_{[5]}$ . It can be easily checked that the pair  $(z, \theta)$  obtained in this case satisfy the null energy condition (NEC) [17]

$$\begin{aligned} (d - \theta)(d(z - 1) - \theta) &\geq 0 \\ (z - 1)(d + z - \theta) &\geq 0 \end{aligned} \quad (4)$$

As the dilaton is not constant, it will produce an RG flow in the boundary theory as  $u$  varies. However, as  $u$  varies, the effective string coupling  $e^\phi$  and the curvature of the metric must remain small for the gravity description to remain valid. This gives a restriction on  $u$  as,  $1/(Q_1 Q_2)^{1/4} \ll u \ll Q_2^{1/12}/Q_1^{1/4}$ . But if  $u \geq Q_2^{1/12}/Q_1^{1/4}$  the dilaton becomes large and we have to uplift the solution to M-theory. The eleven dimensional metric has the form,

$$ds^2 = Q_2^{\frac{2}{3}} \left[ -\frac{2}{Q_2 u^2} dx^{11} dt + \frac{Q_1}{Q_2} u^4 (dx^{11})^2 + \frac{\sum_{i=1}^4 (dx^i)^2}{Q_2 u^2} + 4 \frac{du^2}{u^2} + d\Omega_4^2 \right] \quad (5)$$

The above metric represents an intersecting solution of an M5 brane along  $x^1, \dots, x^4, x^{11}$  with a wave along  $x^{11}$ . This gravity solution is valid as long as  $Q_2 \gg 1$ . From (5) we find that the metric is invariant under an asymmetric Lifshitz scaling  $t \rightarrow \lambda^4 t$ ,  $x^1, \dots, 4 \rightarrow \lambda x^1, \dots, 4$ ,  $x^{11} \rightarrow \lambda^{-2} x^{11}$  and  $u \rightarrow \lambda u$  without any hyperscaling violation ( $\theta = 0$ ).

## 2.2 $p = 2$ or D2-D6 case

It can be seen from the metric in (3) that for  $p = 2$ , the part of the metric in the square bracket is invariant under the scaling  $t \rightarrow \lambda^0 t$ ,  $x^{3,4,5,6} \rightarrow \lambda x^{3,4,5,6}$  and  $u \rightarrow \lambda u$ . Note that the

coordinates  $x^{1,2}$  do not scale. However, the full metric is not invariant as in  $p = 0$  case and therefore there is a hyperscaling violation. In order to find its value we have to compactify the theory on  $S^2 \times T^2$  as the coordinates  $x^{1,2}$  do not scale. The compact theory will therefore be six dimensional and the spatial dimension of the boundary theory is four. Expressing the compact metric in Einstein frame we find that it transforms under the above scaling as  $ds_6 \rightarrow \lambda^{3/2} ds_6 \equiv \lambda^{\theta/d} ds_6$ . We therefore find that the near horizon limit of the intersecting D2-D6 solution has a Lifshitz-like metric with the critical dynamical exponent  $z = 0$  and the hyperscaling violation exponent  $\theta = 6$ . This pair of  $(z, \theta)$  can again be seen to satisfy the NEC (4). The dilaton and the form fields transform under the scaling as  $\phi \rightarrow \phi + \log \lambda$ ,  $A_{[3]} \rightarrow \lambda^2 A_{[3]}$  and  $A_{[7]} \rightarrow \lambda^6 A_{[7]}$ . The dilaton is again found to be non-constant and varies with  $u$  giving an RG flow to the boundary theory. But as the dilaton varies, the effective string coupling  $e^\phi$  and the curvature of the metric must remain small so that the gravity description can be trusted. This gives a restriction on  $u$  as,  $1/(Q_1 Q_2)^{1/4} \ll u \ll Q_2^{3/4}/Q_1^{1/4}$ . When  $u \geq Q_2^{3/4}/Q_1^{1/4}$ , the effective string coupling becomes large and we have to uplift the theory to eleven dimensions. The eleven dimensional metric in this case has the form,

$$ds^2 = Q_1^{1/3} Q_2 u^{4/3} \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_1 Q_2} + \frac{\sum_{j=3}^6 (dx^j)^2}{Q_2 u^2} + 4 \frac{du^2}{u^2} + d\Omega_2^2 + \frac{1}{Q_2^2} (dx^{11} - 2Q_2 \sin^2(\theta/2) d\phi)^2 \right] \quad (6)$$

The solution (6) represents an intersecting solution of M2 branes with KK monopole. Under the scaling  $t \rightarrow \lambda^0 t$ ,  $x^{3,4,5,6} \rightarrow \lambda x^{3,4,5,6}$  and  $u \rightarrow \lambda u$ , the part of the metric in the square bracket remains invariant. Note that  $x^{1,2}$  do not scale. Also, as the whole metric is non-invariant there is a hyperscaling violation. We compactify the theory on  $S^2$  and also along  $x^{1,2,11}$  to obtain the hyperscaling violation exponent. Expressing the reduced metric in the Einstein frame we find that it transforms under the above scaling as,  $ds_6 \rightarrow \lambda^{3/2} ds_6 \equiv \lambda^{\theta/d} ds_6$ , where  $d(= 4)$  is the spatial dimension of the boundary theory. We thus find that the eleven dimensional metric also has a Lifshitz-like structure with dynamical critical exponent  $z = 0$  and hyperscaling violation exponent  $\theta = 6$ . This pair of  $(z, \theta)$  also satisfies the NEC (4).

### 3 $Dp$ -D( $p+2$ ), D2-D2', D3-D3', D4-D4' and Lifshitz-like metrics

In [1], we obtained intersecting D1-D3 solution of type IIB string theory which is 1/4 BPS and threshold bound state. S-dual of this is the F-D3 solution discussed there. D-string in D1-D3 solution is transverse to the D3-brane directions and are delocalized. If we apply T-duality to the common transverse directions of D1-D3 solution we obtain D2-D4 and D3-D5

solution, where D2-D4 intersects on a string and D3-D5 intersects on a membrane. We thus obtain  $Dp$ -D( $p+2$ ) solutions for  $p = 1, 2, 3$ . On the other hand if we apply T-duality along one of the D3-brane directions, we obtain D2-D2' bound state, where the two D2-branes are transverse (intersect on a point) to each other. Further applying T-dualities along the common transverse directions of D2-D2' we can get D3-D3' intersecting on a string and also D4-D4' intersecting on a membrane. Some of these solutions were obtained in [22, 23]. We will show that all these solutions in the near horizon limit give Lifshitz-like metrics. We will also study their phase structures. Since the different solutions have their own peculiarities, we can not study them in generality and therefore discuss each case separately.

### 3.1 D1-D3 case

This case along with its S-dual version have already been discussed in [1] and so we will not repeat it here. We found that both of them have Lifshitz-like structures with  $z = 4$  and  $\theta = 2$ .

### 3.2 D2-D4 case

This configuration can be obtained by applying T-duality along, say,  $x^5$  on the D1-D3 solution given in eq.(5) of ref.[1]. It has the form,

$$\begin{aligned}
ds^2 &= H_1^{\frac{1}{2}} H_2^{\frac{1}{2}} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^3 (dx^i)^2 + H_1^{-1} (dx^4)^2 + H_1^{-1} H_2^{-1} (dx^5)^2 \right. \\
&\quad \left. + dr^2 + r^2 d\Omega_3^2 \right] \\
e^{2\phi} &= \left( \frac{H_1}{H_2} \right)^{\frac{1}{2}} \\
A_{[3]} &= (1 - H_1^{-1}) dt \wedge dx^4 \wedge dx^5, \quad A_{[5]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^5 \quad (7)
\end{aligned}$$

The harmonic functions in (7) are given as,  $H_{1,2} = 1 + Q_{1,2}/r^2$ , with the radial coordinate  $r = \sqrt{(x^6)^2 + \dots + (x^9)^2}$ . It is clear from the metric that D2 branes lie along  $x^4, x^5$ , whereas D4 branes lie along  $x^1, x^2, x^3, x^5$ . In the near horizon limit we approximate  $H_{1,2} \approx Q_{1,2}/r^2$  and then making the coordinate change  $r \rightarrow 1/r$ , the above configuration (7) takes the form,

$$\begin{aligned}
ds^2 &= \sqrt{Q_1 Q_2} \left[ -\frac{dt^2}{Q_1 Q_2 r^2} + \frac{\sum_{i=1}^3 (dx^i)^2}{Q_2} + \frac{(dx^4)^2}{Q_1} + \frac{(dx^5)^2}{Q_1 Q_2 r^2} + \frac{dr^2}{r^2} + d\Omega_3^2 \right] \\
e^{2\phi} &= \left( \frac{Q_1}{Q_2} \right)^{\frac{1}{2}} \\
A_{[3]} &= -\frac{1}{Q_1 r^2} dt \wedge dx^4 \wedge dx^5, \quad A_{[5]} = -\frac{1}{Q_2 r^2} dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^5 \quad (8)
\end{aligned}$$

We thus find that D2-D4 solution does not give Lifshitz-like scaling in the near horizon limit, rather it gives an  $\text{AdS}_3$  space. The gravity description in this case is valid for  $1/Q_2 \ll Q_1 \ll Q_2$ . However, if  $Q_1 > Q_2$ , the effective string coupling would be large and we have to uplift the theory to eleven dimensions. The eleven dimensional metric has the form,

$$ds^2 = Q_1^{\frac{1}{3}} Q_2^{\frac{2}{3}} \left[ -\frac{dt^2}{Q_1 Q_2 r^2} + \frac{\sum_{i=1}^3 (dx^i)^2}{Q_2} + \frac{(dx^4)^2}{Q_1} + \frac{(dx^5)^2}{Q_1 Q_2 r^2} + \frac{(dx^{11})^2}{Q_2} + \frac{dr^2}{r^2} + d\Omega_3^2 \right] \quad (9)$$

The above solution represents the near horizon limit of intersecting M2-M5 branes meeting on a string where M2 branes are along  $x^4$  and  $x^5$ , and M5 branes are along  $x^{1,2,3}$ ,  $x^5$  and  $x^{11}$ . As it is clear this uplifted solution also has  $\text{AdS}_3$  structure. This gravity description can be trusted as long as  $Q_1 \gg 1/Q_2^2$ .

### 3.3 D3-D5 case

This state can be constructed by applying T-duality along one of the common transverse directions ( $x^6$  say) of D2-D4 solution given in (7). The solution takes the form,

$$\begin{aligned} ds^2 &= H_1^{\frac{1}{2}} H_2^{\frac{1}{2}} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^3 (dx^i)^2 + H_1^{-1} (dx^4)^2 + H_1^{-1} H_2^{-1} ((dx^5)^2 + (dx^6)^2) \right. \\ &\quad \left. + dr^2 + r^2 d\Omega_2^2 \right] \\ e^{2\phi} &= \frac{1}{H_2}, \quad F_{[5]} = -(1 + *) dH^{-1} \wedge dt \wedge dx^4 \wedge \dots \wedge dx^6 \\ A_{[6]} &= (1 - H_2^{-1}) dt \wedge dx^1 \wedge \dots \wedge dx^3 \wedge dx^5 \wedge dx^6 \end{aligned} \quad (10)$$

The harmonic functions in this case are given as  $H_{1,2} = 1 + Q_{1,2}/r$ . Here D3-branes lie along  $x^4$ ,  $x^5$ ,  $x^6$  and D5-branes lie along  $x^1$ ,  $x^2$ ,  $x^3$ ,  $x^5$ ,  $x^6$ . Also note that in the above we have given the form of the field-strength (instead of the gauge field) as it is self-dual.

Now taking the near horizon limit  $H_{1,2} \approx Q_{1,2}/r$ , then changing the coordinate  $r$  by  $r \rightarrow 1/r$  and finally, introducing a new coordinate by  $u^2 = 1/r$ , we can rewrite the D3-D5 configuration in the near horizon limit as,

$$\begin{aligned} ds^2 &= \sqrt{Q_1 Q_2} u^2 \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{\sum_{i=1}^3 (dx^i)^2}{Q_2 u^2} + \frac{(dx^4)^2}{Q_1 u^2} + \frac{(dx^5)^2 + (dx^6)^2}{Q_1 Q_2} + 4 \frac{du^2}{u^2} + d\Omega_2^2 \right] \\ e^{2\phi} &= \frac{u^2}{Q_2}, \quad F_{[5]} = -(1 + *) \frac{2u}{Q_1} du \wedge dt \wedge dx^4 \wedge dx^5 \wedge dx^6 \\ A_{[6]} &= -\frac{u^2}{Q_2} dt \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dx^5 \wedge dx^6 \end{aligned} \quad (11)$$

We find from (11) that under the scaling  $t \rightarrow \lambda^0 t$ ,  $x^{1,2,3,4} \rightarrow \lambda x^{1,2,3,4}$ ,  $u \rightarrow \lambda u$ , the part of the metric in the square bracket is invariant, but the full metric is not and so there is

a hyperscaling violation. Note that  $x^{5,6}$  do not scale. To find the hyperscaling violation exponent ( $\theta$ ) we have to compactify the theory on  $S^2$  and also on  $x^{5,6}$  and then express the resulting metric in the Einstein frame. This way we find that the reduced Einstein frame metric transforms under the above scaling as,  $ds_6 \rightarrow \lambda^{3/2} ds_6 \equiv \lambda^{\theta/d} ds_6$ , where  $d = 4$ . Therefore, we get  $\theta = 6$ . We thus find that D3-D5 solution in the near horizon limit has a Lifshitz-like metric with a dynamical critical exponent  $z = 0$  and a hyperscaling violation exponent  $\theta = 6$ . This pair can be shown to satisfy NEC (4). The dilaton and the form fields can be shown to transform under the above scaling as,  $\phi \rightarrow \phi + \log \lambda$ ,  $F_{[5]} \rightarrow \lambda^3 F_{[5]}$  and  $A_{[6]} \rightarrow \lambda^5 A_{[6]}$ .

The gravity description (11) is valid if  $u$  lies in the range  $1/(Q_1 Q_2)^{1/4} \ll u \ll Q_2^{1/2}$ . However for  $u \geq Q_2^{1/2}$ , the effective string coupling  $e^\phi$  becomes large and the gravity description breaks down. For that we have to go to the S-dual frame where the metric and the other fields take the form,

$$\begin{aligned} ds^2 &= Q_1^{\frac{1}{2}} Q_2 u \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{\sum_{i=1}^3 (dx^i)^2}{Q_2 u^2} + \frac{(dx^4)^2}{Q_1 u^2} + \frac{(dx^5)^2 + (dx^6)^2}{Q_1 Q_2} + 4 \frac{du^2}{u^2} + d\Omega_2^2 \right] \\ e^{2\phi} &= \frac{Q_2}{u^2}, \quad F_{[5]} = -(1 + *) \frac{2u}{Q_1} du \wedge dt \wedge dx^4 \wedge dx^5 \wedge dx^6 \\ H_{[3]} &= -Q_2 dx^4 \wedge \epsilon_2 \end{aligned} \tag{12}$$

This represents the near horizon limit of intersecting 1/4 BPS D3-NS5 threshold bound state solution. Under the same scaling as in D3-D5 we find that the part of the metric (see (12)) in square bracket is invariant. However from the transformation of the reduced metric we find that it has a Lifshitz-like structure with the same  $(z, \theta) = (0, 6)$  as in the D3-D5 case. Here the other fields transform as,  $\phi \rightarrow \phi - \log \lambda$ ,  $F_{[5]} \rightarrow \lambda^3 F_{[5]}$  and  $H_{[3]} \rightarrow \lambda H_{[3]}$ .

### 3.4 D2-D2' case

As mentioned earlier, D2-D2' intersecting solution can be obtained by applying T-duality along one of the D3 brane directions (say,  $x^3$ ) of the D1-D3 solution given in eq.(5) of ref.[1]. It has the form,

$$\begin{aligned} ds^2 &= H_1^{\frac{1}{2}} H_2^{\frac{1}{2}} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^2 (dx^i)^2 + H_1^{-1} \sum_{j=3}^4 (dx^j)^2 + dr^2 + r^2 d\Omega_4^2 \right] \\ e^{2\phi} &= (H_1 H_2)^{\frac{1}{2}} \\ A_{[3]} &= (1 - H_1^{-1}) dt \wedge dx^4 \wedge dx^3, \quad A'_{[3]} = (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2 \end{aligned} \tag{13}$$

Here the harmonic functions are given as  $H_{1,2} = 1 + Q_{1,2}/r^3$ . The two D2 branes are along  $x^1, x^2$  and  $x^3, x^4$ . Going to the near horizon limit  $H_{1,2} \approx Q_{1,2}/r^3$ , changing from  $r \rightarrow 1/r$



and introducing a new coordinate by  $u^2 = r$ , we obtain from (13)

$$\begin{aligned}
ds^2 &= (Q_1 Q_2)^{\frac{1}{2}} u^2 \left[ -\frac{dt^2}{Q_1 Q_2 u^8} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_2 u^2} + \frac{\sum_{j=3}^4 (dx^j)^2}{Q_1 u^2} + 4 \frac{du^2}{u^2} + d\Omega_4^2 \right] \\
e^{2\phi} &= (Q_1 Q_2)^{\frac{1}{2}} u^6 \\
A_{[3]} &= -\frac{1}{Q_1 u^6} dt \wedge dx^4 \wedge dx^3, \quad A'_{[3]} = -\frac{1}{Q_2 u^6} dt \wedge dx^1 \wedge dx^2
\end{aligned} \tag{14}$$

Thus we find that under the scaling  $t \rightarrow \lambda^4 t$ ,  $x^{1,2,3,4} \rightarrow \lambda x^{1,2,3,4}$  and  $u \rightarrow \lambda u$ , the part of the metric (given in (14)) in the square bracket remains invariant. But the whole metric is not invariant because of the hyperscaling violation. As before, we find that the reduced metric in the Einstein frame transforms under the above scaling as  $ds_6 \rightarrow \lambda^{1/2} ds_6 \equiv \lambda^{\theta/d} ds_6$ . We thus find  $\theta = 2$ . Therefore, D2-D2' solution in the near horizon limit has Lifshitz-like metric with  $z = 4$  and  $\theta = 2$ . This pair of  $(z, \theta)$  can be shown to satisfy NEC (4). The dilaton transforms under the scaling as,  $\phi \rightarrow \phi + 3 \log \lambda$ .  $A_{[3]}$  and  $A'_{[3]}$  remain invariant.

In this case the above gravity description is valid for  $1/(Q_1 Q_2)^{1/4} \ll u \ll 1/(Q_1 Q_2)^{1/12}$ . When  $u \geq 1/(Q_1 Q_2)^{1/12}$ , the effective string coupling  $e^\phi$  becomes large and the gravity description breaks down. In that case we have to uplift the solution to M-theory. The uplifted solution has the form,

$$ds^2 = (Q_1 Q_2)^{\frac{1}{3}} u^{\frac{2}{3}} \left[ -\frac{dt^2}{Q_1 Q_2 u^6} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_2 u^2} + \frac{\sum_{j=3}^4 (dx^j)^2}{Q_1 u^2} + \frac{du^2}{u^2} + d\Omega_5^2 \right] \tag{15}$$

This represents two intersecting M2 branes along  $x^1, x^2$  and  $x^3, x^4$ . The part of the metric in the square bracket has the scale invariance  $t \rightarrow \lambda^3 t$ ,  $x^{1,2,3,4} \rightarrow \lambda x^{1,2,3,4}$ ,  $u \rightarrow \lambda u$ . The metric has a Lifshitz-like structure with  $(z, \theta) = (3, 3)$ .

### 3.5 D3-D3' case

This bound state can be obtained by applying T-duality along one of the common transverse directions ( $x^5$ , say) of the D2-D2' solution given in (13). This way we obtain D3-D3' solution in the following form,

$$\begin{aligned}
ds^2 &= H_1^{\frac{1}{2}} H_2^{\frac{1}{2}} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^2 (dx^i)^2 + H_1^{-1} \sum_{j=3}^4 (dx^j)^2 + H_1^{-1} H_2^{-1} (dx^5)^2 \right. \\
&\quad \left. + dr^2 + r^2 d\Omega_3^2 \right] \\
e^{2\phi} &= 1 \\
F_{[5]} &= -(1 + *) dH_1^{-1} \wedge dt \wedge dx^4 \wedge dx^3 \wedge dx^5 \\
F'_{[5]} &= -(1 + *) dH_2^{-1} \wedge dt \wedge dx^1 \wedge dx^2 \wedge dx^5
\end{aligned} \tag{16}$$

Here the two D3-branes are along the directions  $x^1, x^2, x^5$  and  $x^3, x^4, x^5$ , i.e., they intersect on a string. The harmonic functions are given as  $H_{1,2} = 1 + Q_{1,2}/r^2$ .  $F_{[5]}$  and  $F'_{[5]}$  are the two self-dual field-strengths to which the D3-branes couple. In the near horizon limit  $H \approx Q_{1,2}/r^2$ , along with the change of coordinates  $r \rightarrow 1/r$ , the configuration (16) takes the form,

$$\begin{aligned}
ds^2 &= (Q_1 Q_2)^{\frac{1}{2}} \left[ -\frac{dt^2}{Q_1 Q_2 r^2} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_2} + \frac{\sum_{j=3}^4 (dx^j)^2}{Q_1} + \frac{(dx^5)^2}{Q_1 Q_2 r^2} + \frac{dr^2}{r^2} + d\Omega_3^2 \right] \\
e^{2\phi} &= 1 \\
F_{[5]} &= (1 + *) \frac{2}{Q_1 r^3} dr \wedge dt \wedge dx^4 \wedge dx^3 \wedge dx^5 \\
F'_{[5]} &= (1 + *) \frac{2}{Q_2 r^3} dr \wedge dt \wedge dx^1 \wedge dx^2 \wedge dx^5
\end{aligned} \tag{17}$$

It is clear from the above, that D3-D3' indeed has  $\text{AdS}_3$  structure in the near horizon limit. Note that the supergravity description is valid as long as the string coupling  $g_s$  (which is suppressed here) is small and  $Q_1 \gg 1/Q_2$ .

### 3.6 D4-D4' case

The application of a further T-duality along a common transverse direction ( $x^6$ , say) of D3-D3' solution given in (16) will produce D4-D4' solution given as,

$$\begin{aligned}
ds^2 &= H_1^{\frac{1}{2}} H_2^{\frac{1}{2}} \left[ -H_1^{-1} H_2^{-1} dt^2 + H_2^{-1} \sum_{i=1}^2 (dx^i)^2 + H_1^{-1} \sum_{j=3}^4 (dx^j)^2 \right. \\
&\quad \left. + H_1^{-1} H_2^{-1} \sum_{k=5}^6 (dx^k)^2 + dr^2 + r^2 d\Omega_2^2 \right] \\
e^{2\phi} &= (H_1 H_2)^{-\frac{1}{2}} \\
A_{[5]} &= (1 - H_1^{-1}) dt \wedge dx^4 \wedge dx^3 \wedge dx^5 \wedge dx^6 \\
A'_{[5]} &= (1 - H_2^{-1}) dt \wedge dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^6
\end{aligned} \tag{18}$$

From (18) we observe that the two D4-branes in this solution lie along  $x^1, x^2, x^5, x^6$  and  $x^3, x^4, x^5, x^6$ , i.e., they intersect on a membrane. The harmonic functions are given as  $H_{1,2} = 1 + Q_{1,2}/r$ . Taking the near horizon limit  $H_{1,2} \approx Q_{1,2}/r$ , changing coordinates  $r \rightarrow 1/r$  and introducing new variable by  $u^2 = 1/r$ , we rewrite the above solution in terms

of this new variable as,

$$\begin{aligned}
ds^2 &= (Q_1 Q_2)^{\frac{1}{2}} u^2 \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_2 u^2} + \frac{\sum_{j=3}^4 (dx^j)^2}{Q_1 u^2} + \frac{\sum_{k=5}^6 (dx^k)^2}{Q_1 Q_2} + 4 \frac{du^2}{u^2} + d\Omega_2^2 \right] \\
e^{2\phi} &= \frac{u^2}{(Q_1 Q_2)^{\frac{1}{2}}} \\
A_{[5]} &= -\frac{u^2}{Q_1} dt \wedge dx^4 \wedge dx^3 \wedge dx^5 \wedge dx^6 \\
A'_{[5]} &= -\frac{u^2}{Q_2} dt \wedge dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^6
\end{aligned} \tag{19}$$

We observe from (19) that the part of the metric in square bracket is invariant under the scaling  $t \rightarrow \lambda^0 t$ ,  $x^{1,2,3,4} \rightarrow \lambda x^{1,2,3,4}$ ,  $u \rightarrow \lambda u$ . However, the full metric is not invariant under this scaling and so there is a hyperscaling violation. Compactifying the theory on  $S^2$  and also along  $x^{5,6}$  (as they do not scale) we find that the reduced metric in Einstein frame transforms as  $ds_6 \rightarrow \lambda^{3/2} ds_6 \equiv \lambda^{\theta/d} ds_6$ , where  $d(=4)$  is the spatial dimension of the boundary theory. This gives  $\theta = 6$ . We therefore find that D4-D4' solution in the near horizon limit has a Lifshitz-like metric with  $z = 0$  and  $\theta = 6$ . As before this pair ( $z = 0$ ,  $\theta = 6$ ) satisfies NEC (4). The dilaton and the form fields transform under the above scaling as  $\phi \rightarrow \phi + \log \lambda$ ,  $A_{[5]} \rightarrow \lambda^4 A_{[5]}$  and  $A'_{[5]} \rightarrow \lambda^4 A'_{[5]}$ .

We note that the above gravity description is valid when the effective string coupling  $e^\phi$  and the curvature of the metric remain small. This gives the restriction on  $u$  as  $1/(Q_1 Q_2)^{1/4} \ll u \ll (Q_1 Q_2)^{1/4}$ . However when  $u \geq (Q_1 Q_2)^{1/4}$  we have to uplift the solution to M-theory. The eleven dimensional solution can be seen to take the form,

$$\begin{aligned}
ds^2 &= (Q_1 Q_2)^{\frac{2}{3}} u^{\frac{4}{3}} \left[ -\frac{dt^2}{Q_1 Q_2} + \frac{\sum_{i=1}^2 (dx^i)^2}{Q_2 u^2} + \frac{\sum_{j=3}^4 (dx^j)^2}{Q_1 u^2} + \frac{\sum_{k=5}^6 (dx^k)^2}{Q_1 Q_2} + 4 \frac{du^2}{u^2} + d\Omega_2^2 \right. \\
&\quad \left. + \frac{(dx^{11})^2}{Q_1 Q_2} \right] \\
A_{[6]} &= -\frac{u^2}{Q_1} dt \wedge dx^4 \wedge dx^3 \wedge dx^5 \wedge dx^6 \wedge dx^{11} \\
A'_{[6]} &= -\frac{u^2}{Q_2} dt \wedge dx^1 \wedge dx^2 \wedge dx^5 \wedge dx^6 \wedge dx^{11}
\end{aligned} \tag{20}$$

This solution represents two M5 branes intersecting on a three brane along  $x^5$ ,  $x^6$  and  $x^{11}$ . We again find that the metric has Lifshitz-like scaling as the part of the metric in the square bracket is invariant under same scaling as the D4-D4' solution. Again as the full metric is not scale invariant, there is a hyperscaling violation. The hyperscaling violation exponent can be found as before by reducing the metric on  $S^2$  and also on  $x^{5,6,11}$  and expressing the resulting metric in Einstein frame. We thus find that  $\theta$  has the value 6. Therefore, D4-D4' solution in the strong coupling phase also has Lifshitz-like scaling with  $z = 0$  and  $\theta = 6$ .

## 4 Conclusion

To conclude, in this paper we have shown how Lifshitz-like metrics (space-time metrics having Lifshitz scaling with hyperscaling violation) arise from the near horizon limit of certain intersecting D-brane solutions of type II string theories. Some of these solutions are standard 1/4 BPS D0-D4, D2-D6 threshold bound states and some are non-standard 1/4 BPS D1-D3, D2-D4, D3-D5 along with D2-D2', D3-D3', D4-D4' threshold bound states. All these solutions (except D2-D4 and D3-D3') in the near horizon limit gave rise to Lifshitz-like metrics with some dynamical critical exponent  $z$  and some hyperscaling violation exponent  $\theta$ . D2-D4 and D3-D3' solutions gave  $\text{AdS}_3$  spaces. We found that in all these solutions except D2-D4 and D3-D3', the dilatons were non-constant. We discussed also the phase structures of various solutions. The metrics in other phases were also found to have Lifshitz-like structures. For the various solutions, we found that there are two sets of values for the dynamical critical exponent ( $z$ ), the hyperscaling violation exponent ( $\theta$ ) and the spatial dimension ( $d$ ) of the boundary theory. The solutions D2-D6, D3-D5 and D4-D4' as well as their strongly coupled phases yielded Lifshitz-like metrics in the near horizon limit with  $(z = 0, \theta = 6, d = 4)$ . On the other hand, the solutions D0-D4, D1-D3 (and its strongly coupled phase) and D2-D2' yielded Lifshitz-like metrics in the near horizon limit with  $(z = 4, \theta = 2, d = 4)$ . We have checked that both these values satisfy null energy condition. However, none of these values satisfy  $\theta = d - 1$ . As emphasized in [15, 16] that theories of this type may be of interest to some condensed matter system such as compressible metallic states with hidden Fermi surface. Whether theories corresponding to the values of  $z$  and  $\theta$  we have obtained in this paper have any potential application to condensed matter system is yet to be seen.

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